Lecture-2

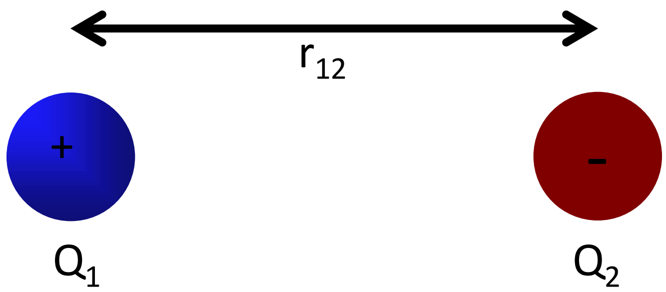
Field intensity,
Electric field due to charge distribution, Electric flux density

,

Coulomb's Law:
The Big Picture

Coulomb's Law quantifies the interaction between charged particles.

$$F_{12} = \frac{1}{4\pi\varepsilon_0} \frac{|q_1 q_2|}{r_{12}^2},$$



Coulomb's Law was discovered through decades of experiment.

The Electric Field

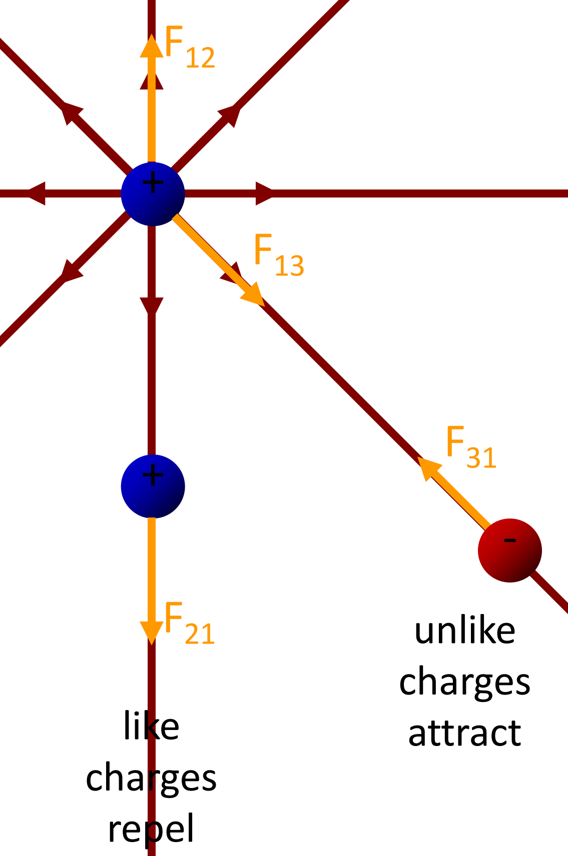
Coulomb's Law (demonstrated in 1785) shows that charged particles exert forces on each other over great distances.

How does a charged particle "know" another one is “there?”

Faraday, beginning in the 1830's, was the leader in developing the idea of the electric field. Here's the idea:

- A charged particle emanates a "field" into all space.

- Another charged particle senses the field, and “knows” that the first one is there.



We define the electric field by the force it exerts on a test charge q_0:

$$\vec{E} = \frac{\vec{F}_0}{q_0}$$

If the test charge is "too big" it perturbs the electric field, so the “correct” definition is

$$\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}_0}{q_0}$$

You won’t be required to use this version of the equation.

Any time you know the electric field, you can use this equation to calculate the force on a charged particle in that electric field. $\vec{F} = q\vec{E}$

The units of electric field are Newtons/Coulomb.

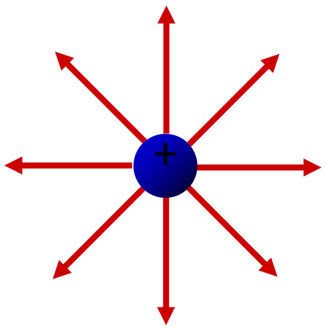
$$\left[\vec{E}\right] = \frac{\left[\vec{F}_0\right]}{\left[q_0\right]} = \frac{N}{C}$$

Later you will learn that the units of electric field can also be expressed as volts/meter:

$$\left[E\right] = \frac{N}{C} = \frac{V}{m}$$

The electric field exists independent of whether there is a charged particle around to "feel" it.

Remember: the electric field direction is the direction a + charge would feel a force.



A + charge would be repelled by another + charge.

Therefore the direction of the electric field is away from positive (and towards negative).

The Electric Field
Due to a Point Charge

Coulomb's law says

$$F_{12} = k\frac{|q_1 q_2|}{r_{12}^2},$$

... which tells us the electric field due to a point charge q is

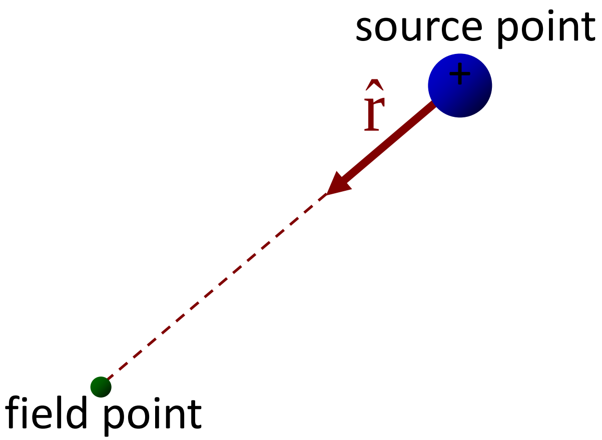
$$\vec{E}_q = k\frac{|q|}{r^2} \text{, away from +}$$

...or just...

$$E = k\frac{|q|}{r^2}$$

This is your third starting equation.

We define $\hat{r}$ as a unit vector from the source point to the field point:



The equation for the electric field of a point charge then becomes:

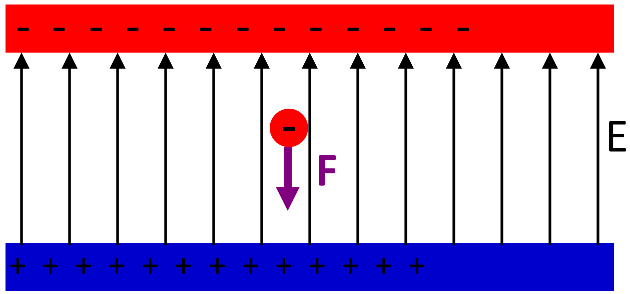
$$\vec{E} = k\frac{q}{r^2}\hat{r}$$

Motion of a Charged Particle in a Uniform Electric Field

A charged particle in an electric field experiences a force, and if it is free to move, an acceleration.

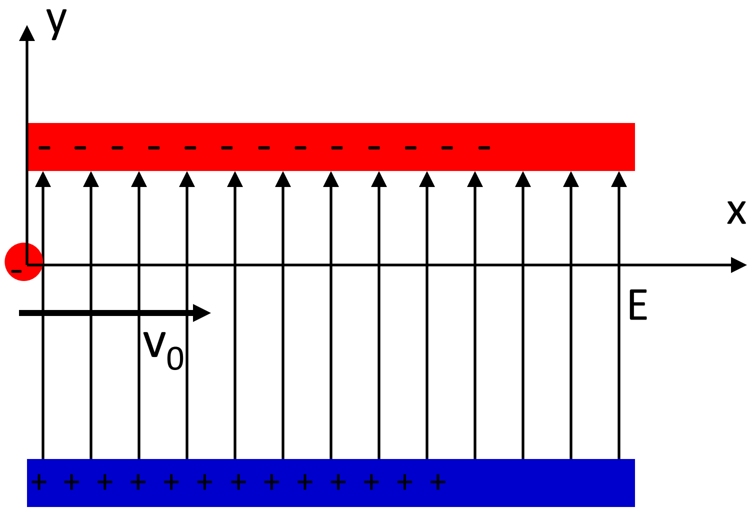
If the only force is due to the electric field, then

$$\sum \vec{F} = m\vec{a} = q\vec{E}.$$



If $\vec{E}$ is constant, then $\vec{a}$ is constant, and you can use the equations of kinematics.

Example: an electron moving with velocity v_0 in the positive x direction enters a region of uniform electric field that makes a right angle with the electron's initial velocity. Express the position and velocity of the electron as a function of time.



The Electric Field
Due to a Collection of Point Charges

The electric field due to a small "chunk" Δq of charge is

$$\Delta\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{\Delta q}{r^2}\hat{r}$$

unit vector from Δq to wherever you want to calculate $\Delta\vec{E}$

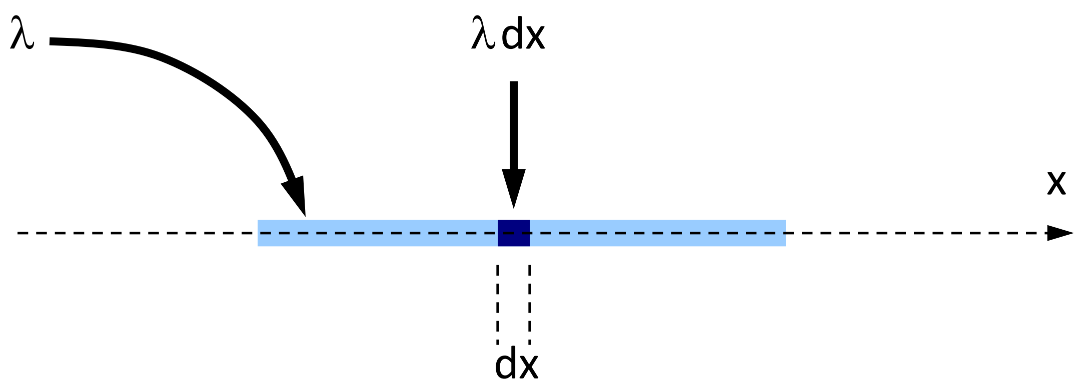
The electric field due to collection of "chunks" of charge is

$$\vec{E} = \sum_i \Delta\vec{E}_i = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{\Delta q_i}{r_i^2}\hat{r}_i$$

As $\Delta q \rightarrow dq \rightarrow 0$, the sum becomes an integral.

unit vector from Δq_i to wherever you want to calculate $\vec{E}$

If charge is distributed along a straight line segment parallel to the x-axis, the amount of charge dq on a segment of length dx is λdx.

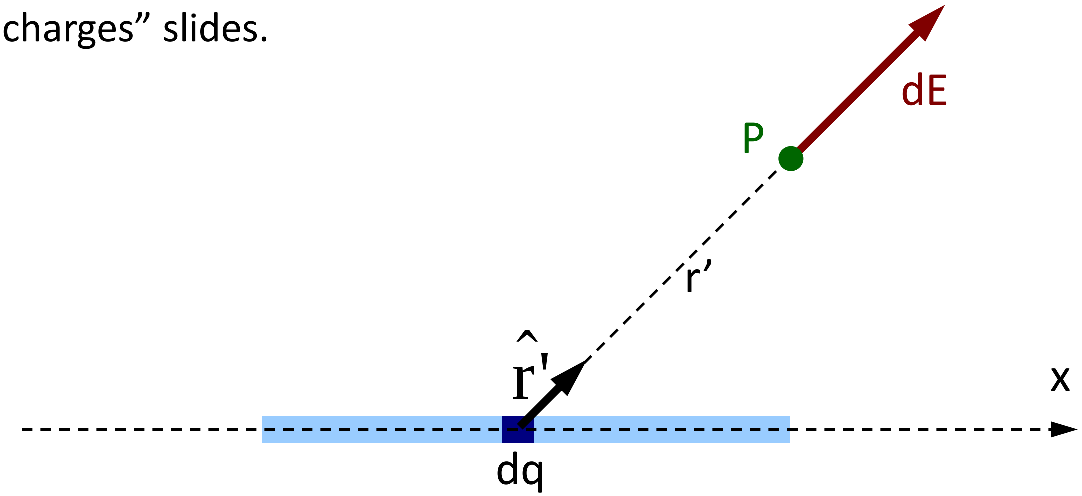


λ is the linear density of charge (amount of charge per unit length). λ may be a function of position.

Think $\lambda \Leftrightarrow$ $\Leftrightarrow$ length. λ times the length of line segment is the total charge on the line segment.

ℓ

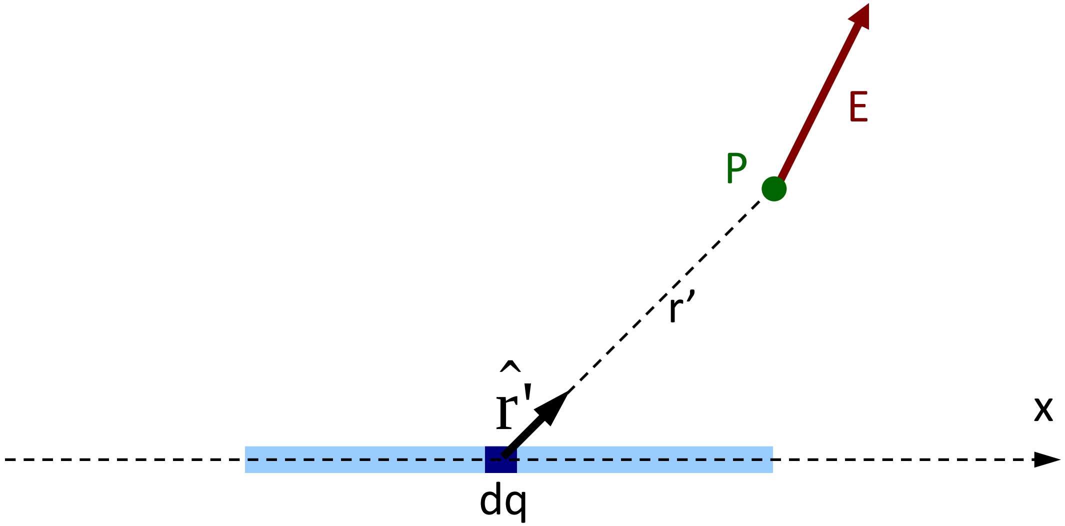
I’m assuming positively charged objects in these “distribution of charges” slides.



The electric field at point P due to the charge dq is

$$d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r'^2}\hat{r}' = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dx}{r'^2}\hat{r}'$$

$$dE = k\frac{|dq|}{r^2}$$

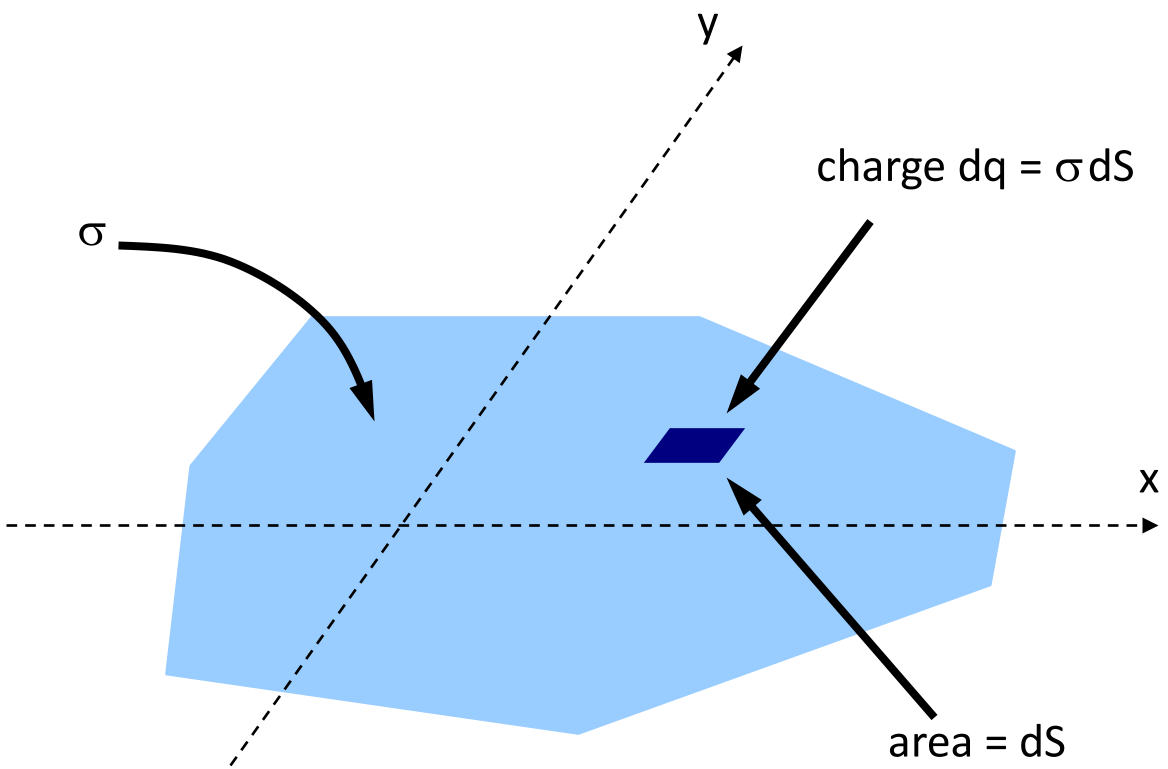


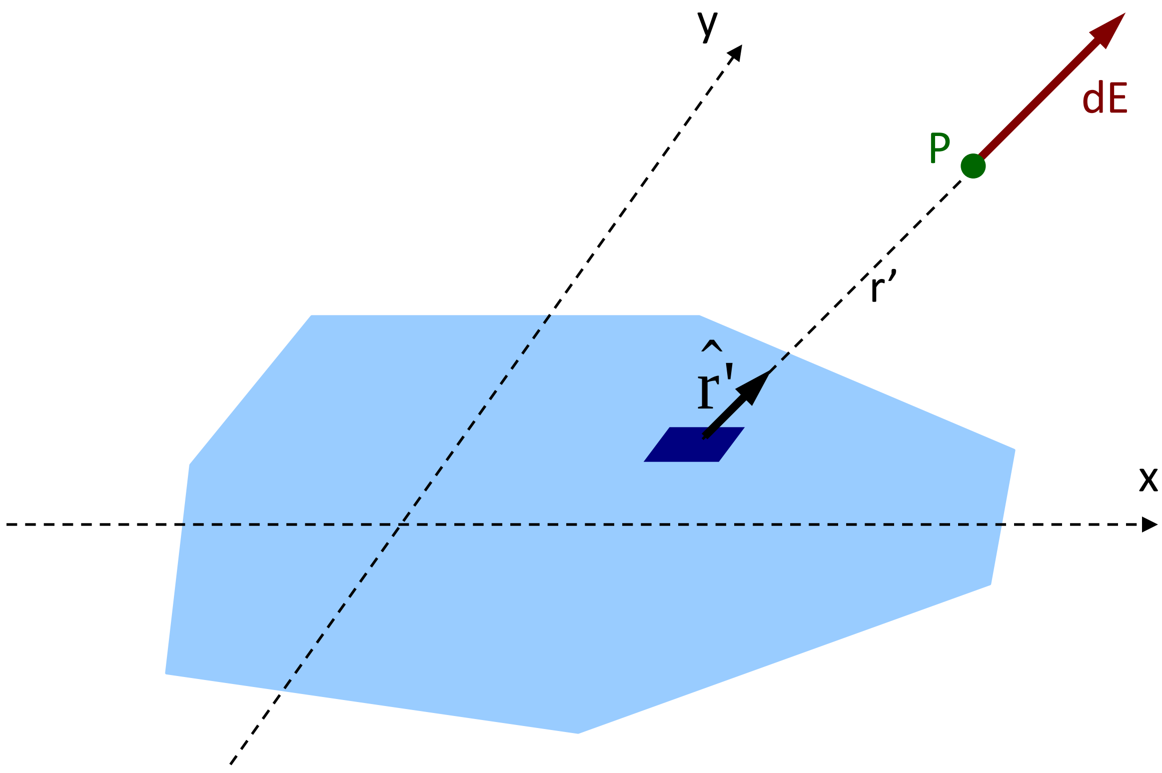
The electric field at P due to the entire line of charge is

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int \hat{r}'\frac{\lambda(x)\,dx}{r'^2}.$$

The integration is carried out over the entire length of the line, which need not be straight. Also, λ could be a function of position, ***and can be taken outside the integral only if the charge distribution is uniform***.

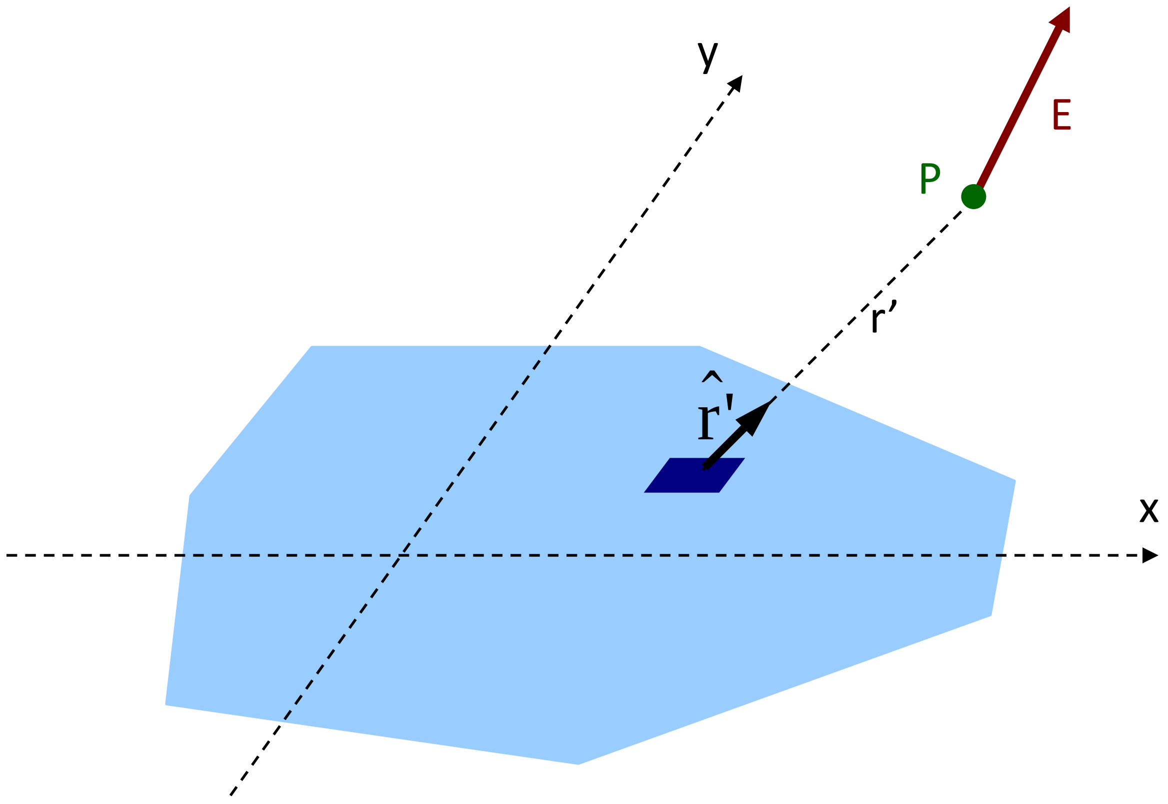
If charge is distributed over a two-dimensional surface, the amount of charge dq on an infinitesimal piece of the surface is σdS, where σ is the surface density of charge (amount of charge per unit area).





The electric field at P due to the charge dq is

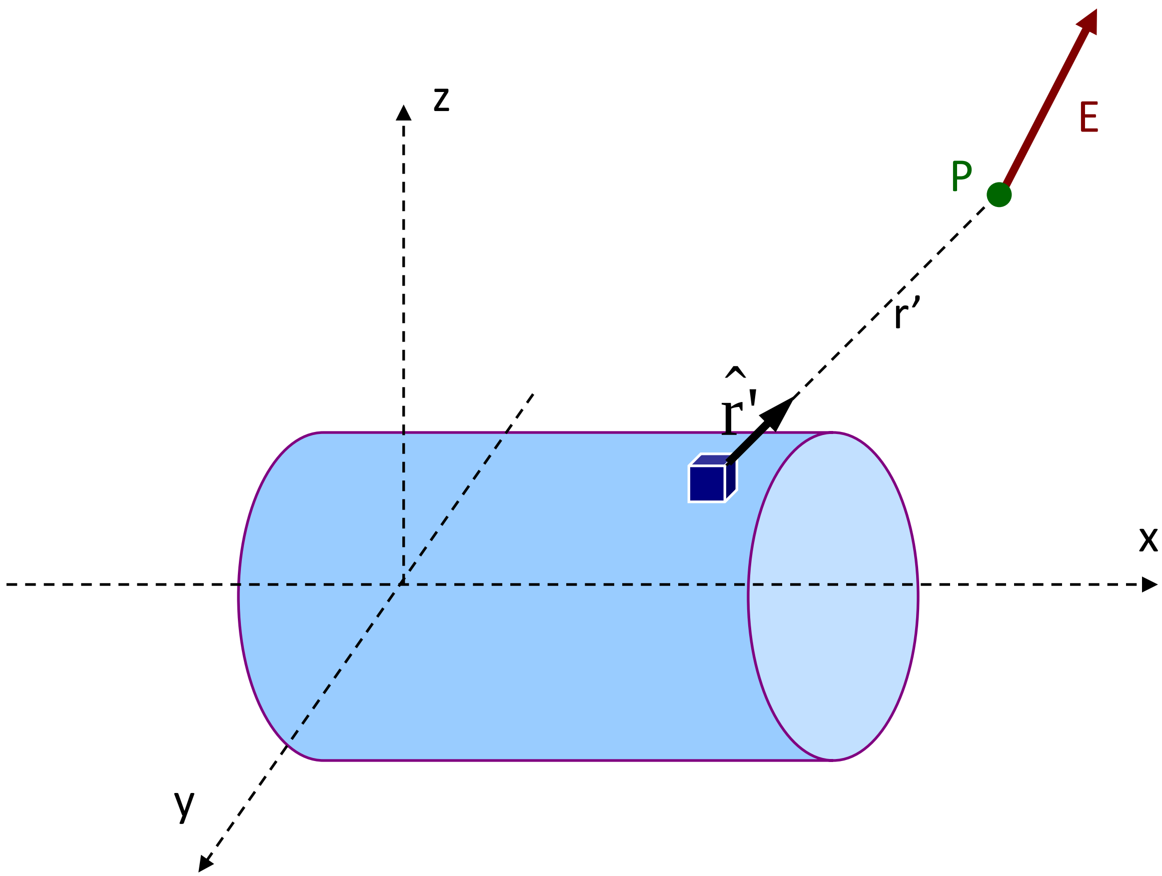
$$d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r'^2}\hat{r}' = \frac{1}{4\pi\varepsilon_0}\frac{\sigma\ dS}{r'^2}\hat{r}'$$



The net electric field at P due to the entire surface of charge is

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int_S \hat{r}'\frac{\sigma(x,y)\, dS}{r'^2}$$

After you have seen the above, I hope you believe that the net electric field at P due to a three-dimensional distribution of charge is…



$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int_V \hat{r}'\frac{\rho(x,y,z)\,dV}{r'^2}$$

Summarizing:

Charge distributed along a line:

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int \hat{r}' \frac{\lambda\, dx}{r'^2}.$$

Charge distributed over a surface:

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int_S \hat{r}' \frac{\sigma\, dS}{r'^2}.$$

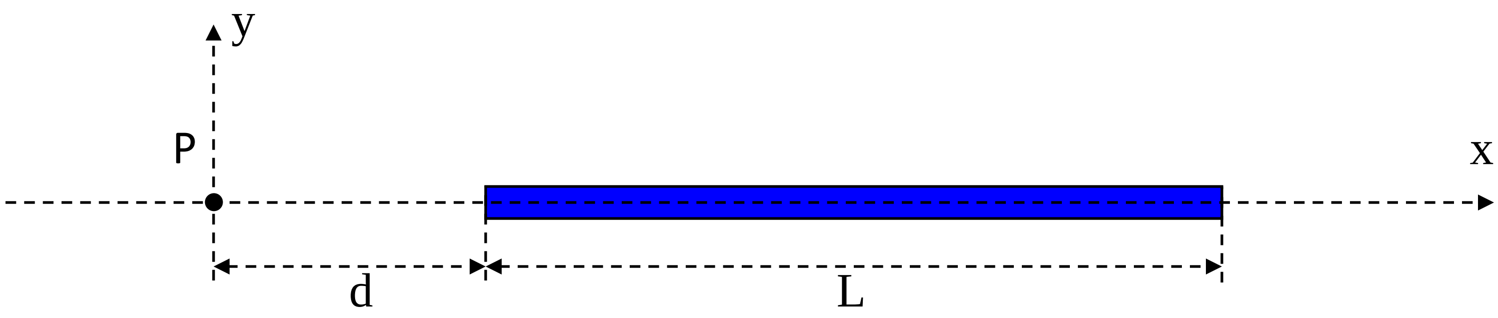
Charge distributed inside a volume:

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int_V \hat{r}' \frac{\rho\, dV}{r'^2}.$$

If the charge distribution is uniform, then λ, σ, and ρ can be taken outside the integrals.

The Electric Field Due to a Continuous Charge Distribution (worked examples)

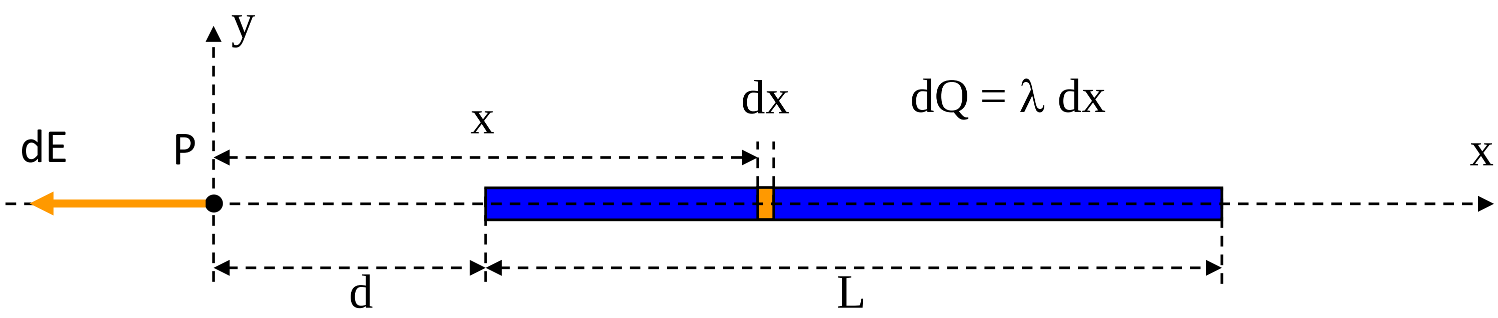
Example: A rod of length L has a uniform charge per unit length λ and a total charge Q. Calculate the electric field at a point P along the axis of the rod at a distance d from one end.



Let's put the origin at P. The linear charge density and Q are related by

$$\lambda = \frac{Q}{L} \quad \text{and} \quad Q = \lambda L$$

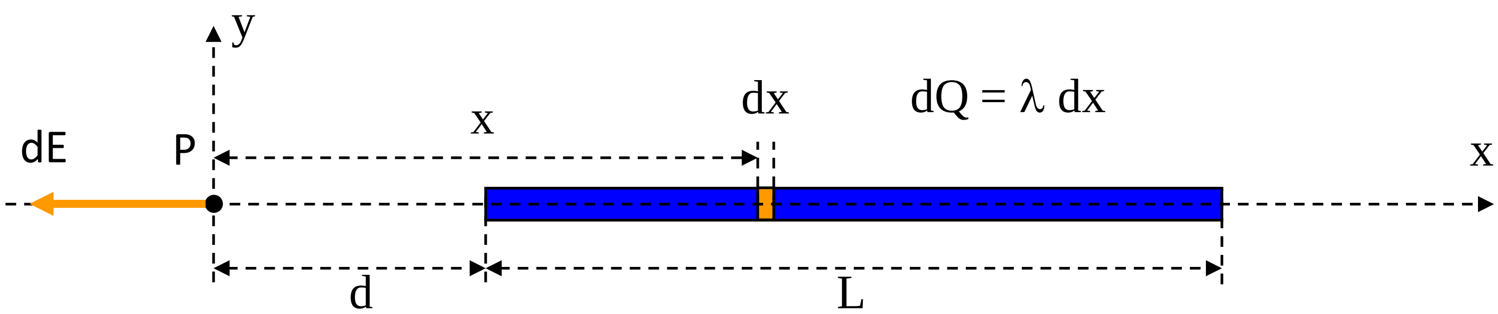
Let's assume Q is positive.



The electric field points away from the rod. By symmetry, the electric field on the axis of the rod has no y-component. dE from the charge on an infinitesimal length dx of rod is

$$dE = k\frac{dq}{x^2} = k\frac{\lambda\ dx}{x^2}$$

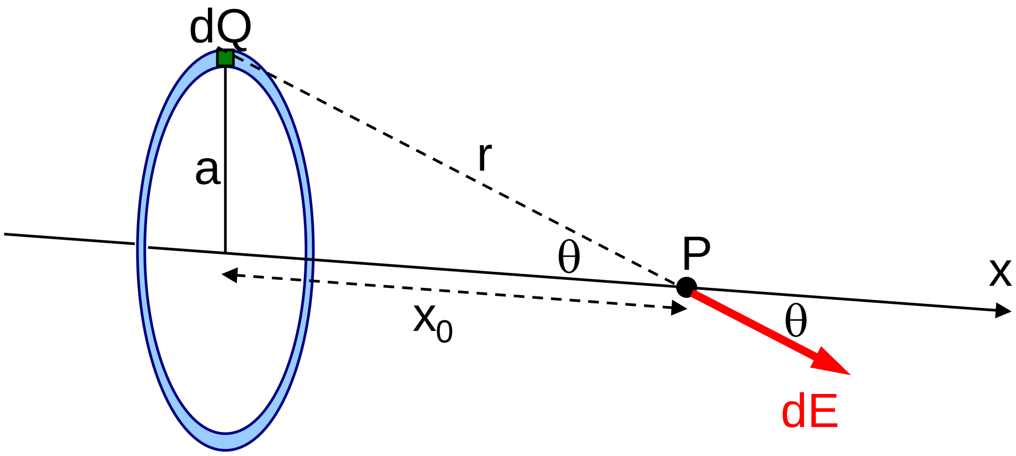
Note: $\vec{dE}$ is in the –x direction. dE is the magnitude of $\vec{dE}$. I've used the fact that Q>0 (so dq>0) to eliminate the absolute value signs in the starting equation.



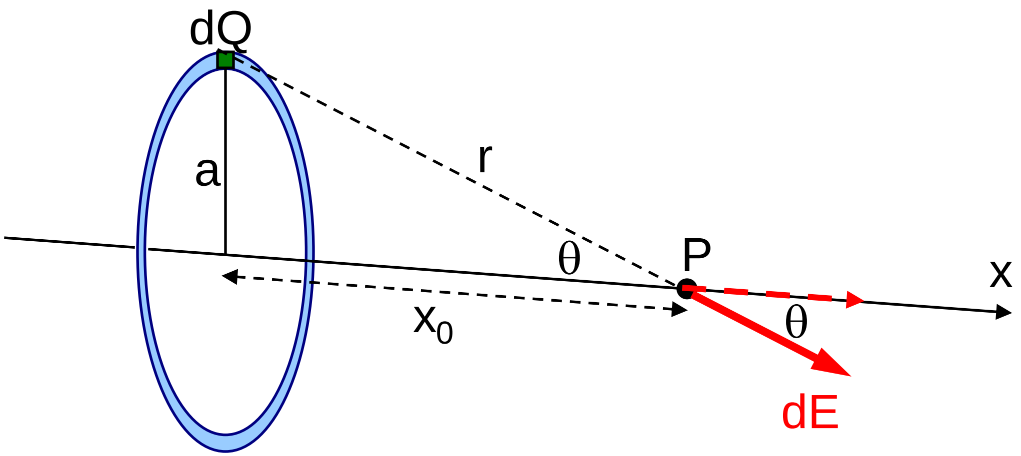
$$\vec{E} = \int_d^{d+L} d\vec{E}_x = -k\int_d^{d+L} \frac{\lambda\ dx}{x^2}\hat{i} = -k\lambda\int_d^{d+L} \frac{dx}{x^2}\hat{i} = -k\lambda\left(-\frac{1}{x}\right)_d^{d+L}\hat{i}$$

$$\vec{E} = -k\lambda\left(-\frac{1}{d+L}+\frac{1}{d}\right)\hat{i} = -k\lambda\left(\frac{-d+d+L}{d(d+L)}\right)\hat{i} = -k\frac{\lambda\ L}{d(d+L)}\hat{i} = -\frac{kQ}{d(d+L)}\hat{i}$$

Example: A ring of radius a has a uniform charge per unit length and a total positive charge Q. Calculate the electric field at a point P along the axis of the ring at a distance x_0 from its center.



By symmetry, the y- and z-components of $\vec{E}$ are zero, and all points on the ring are a distance r from point P.



$$dE = k\frac{dQ}{r^2}$$

No absolute value signs because Q is positive.

$$dE_x = k\frac{dQ}{r^2}\cos\theta$$

$$r = \sqrt{x_0^2 + a^2} \qquad \cos\theta = \frac{x_0}{r}$$

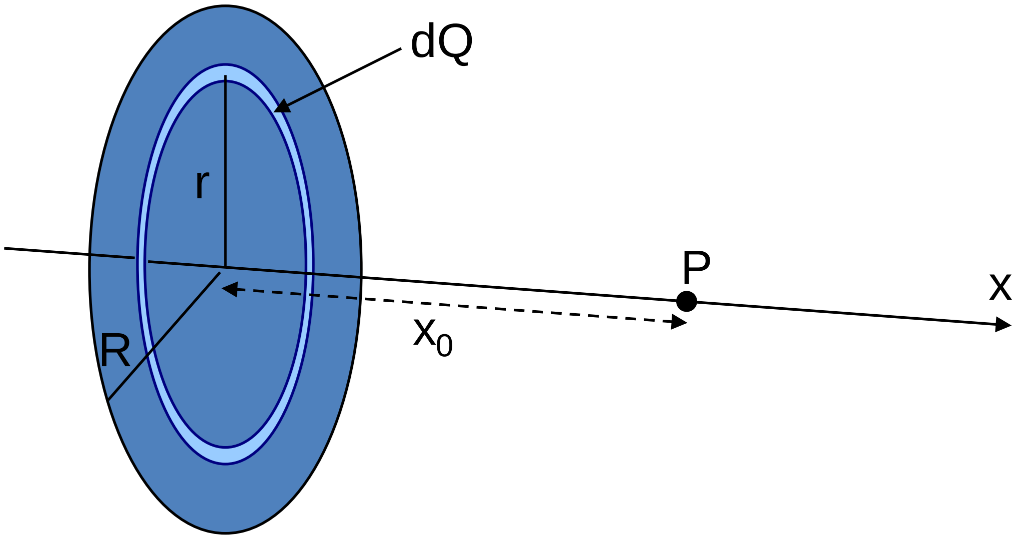
For a given x_0, r is a constant for points on the ring.

$$E_x = \int_{ring} dE_x = \int_{ring}\left(k\frac{dQ}{r^2}\right)\frac{x_0}{r} = k\frac{x_0}{r^3}\int_{ring} dQ = k\frac{x_0}{r^3}Q = \frac{kx_0Q}{\left(x_0^2 + a^2\right)^{3/2}}$$

Or, in general, on the ring axis

$$E_{x,ring} = \frac{kxQ}{\left(x^2 + a^2\right)^{3/2}}.$$

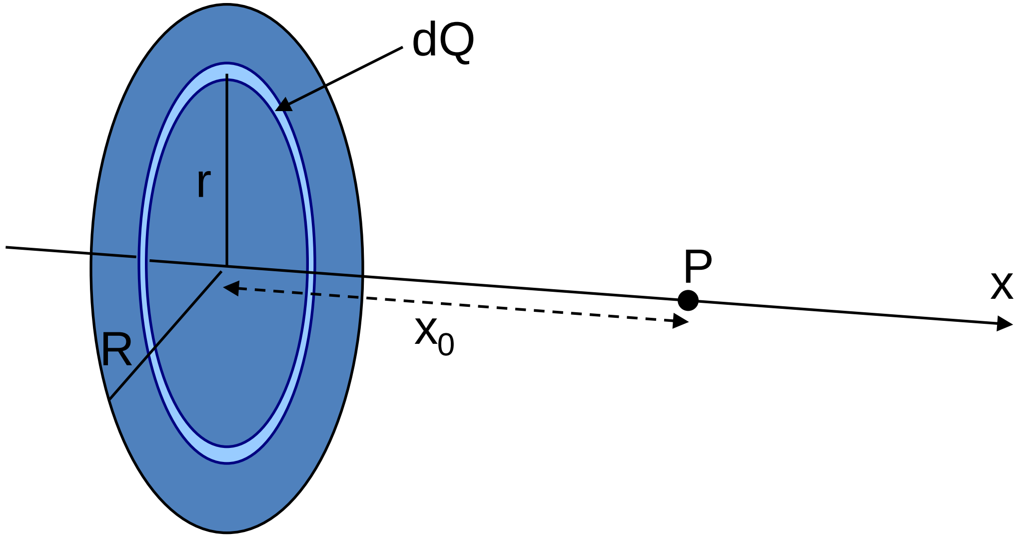
Example: A disc of radius R has a uniform charge per unit area σ. Calculate the electric field at a point P along the central axis of the disc at a distance x_0 from its center.



The disc is made of concentric rings. The area of a ring at a radius r is $2\pi r dr$, and the charge on each ring is $\sigma(2\pi r dr)$.

We can use the equation on the previous slide for the electric field due to a ring, replace a by r, and integrate from r=0 to r=R.

$$dE_{ring} = \frac{k x_0 \sigma 2\pi r dr}{\left(x_0^2 + r^2\right)^{3/2}}.$$



$$E_x = \int_{disc} dE_x = \int_{disc} \frac{kx_0\sigma 2\pi r dr}{\left(x_0^2 + r^2\right)^{3/2}} = kx_0\pi\sigma \int_0^R \frac{2r\ dr}{\left(x_0^2 + r^2\right)^{3/2}}$$

$$E_x = kx_0\pi\sigma \left[\frac{\left(x_0^2 + r^2\right)^{-1/2}}{-1/2} \right]_0^R = 2k\pi\sigma \left(\frac{x_0}{|x_0|} - \frac{x_0}{\left(x_0^2 + R^2\right)^{1/2}} \right)$$

Example: Calculate the electric field at a distance x_0 from an infinite plane sheet with a uniform charge density σ.

Treat the infinite sheet as disc of infinite radius.

Let $R \to \infty$ and use to get $k = \frac{1}{4\pi\varepsilon_0}$

$$E_{sheet} = \frac{|\sigma|}{2\varepsilon_0}.$$

Interesting...does not depend on distance from the sheet.